ACC 2025

Task Hierarchical Control via Null-Space Projection and Path Integral Approach

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Outline

What is Task Hierarchical Control?

Null-Space Projection

Proposed Approach

Simulation Results

Future Work

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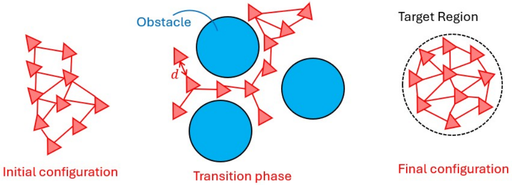

- Drone swarm: A Large degrees of freedom system

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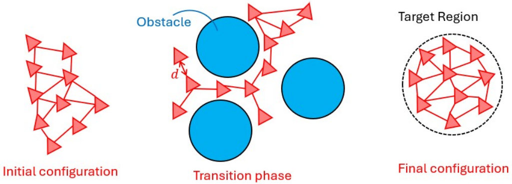

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- Three goals to accomplish in descending order of importance:
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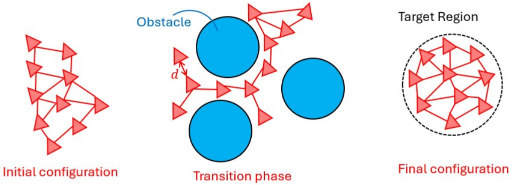

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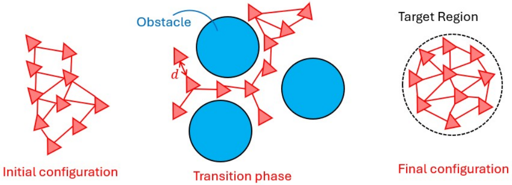

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How to achieve this?

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Null-Space Projection

- The top priority task is executed by employing all the capabilities of the system
- The second priority task is then applied to the null space of the top priority task
- The task on level three is then executed without disturbing the two higher-priority tasks, and so forth ...

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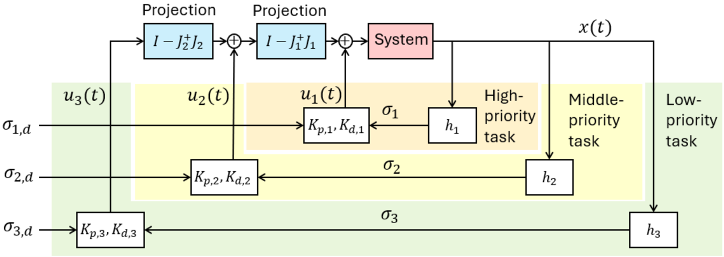
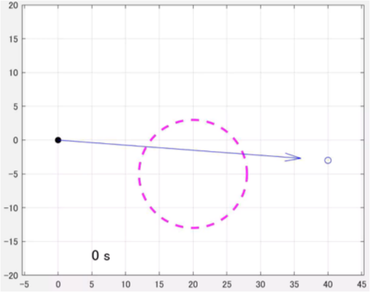
$$\dot{\sigma}_k = J_k(q)\dot{q}, \qquad J_k(q) = \frac{\partial h_k(q)}{\partial q}$$

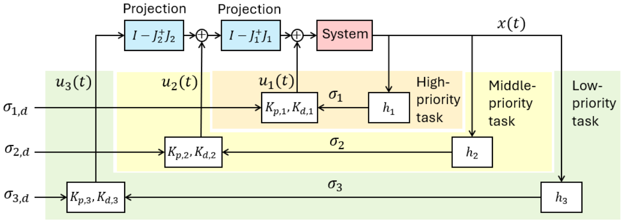


An Example



- Task 1: Avoid collision with the obstacle
- Task 2: Reach the goal

Current Limitations



- The individual controllers for the tasks are designed using simple low-level controllers such as PID controllers

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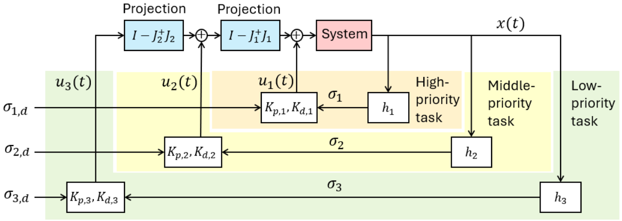

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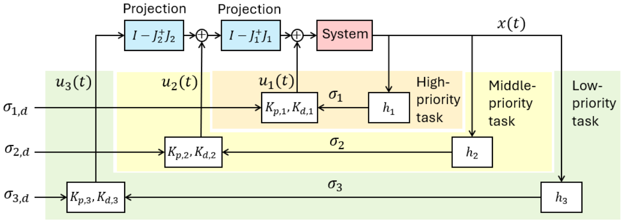

- The individual controllers for the tasks are designed using simple low-level controllers such as PID controllers
- Desired signals $\sigma_{i,d}(t)$ and control gains $K_{p,i}$, $K_{d,i}$ are often chosen manually
- There is no systematic way to optimize the overall performance of the hierarchy of controllers

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- We introduce a new framework for hierarchical task control that combines the null-space projection technique with the path integral control method

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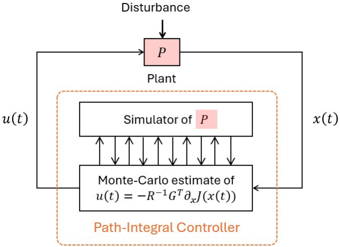
- This leverages Monte Carlo simulations for real-time computation of optimal control inputs
- This allows for the seamless integration of simpler PID-like controllers with a more sophisticated optimal control technique

What is Path Integral Control?



- A control algorithm inspired by the path integral formulation of quantum mechanics

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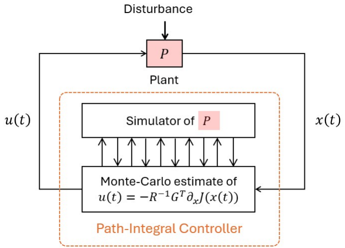

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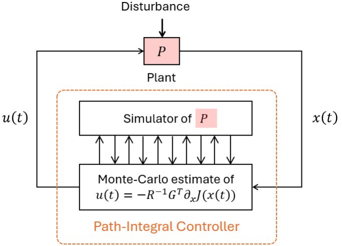

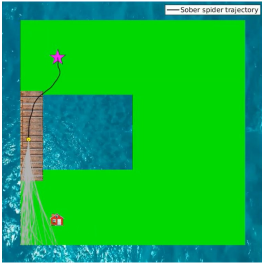

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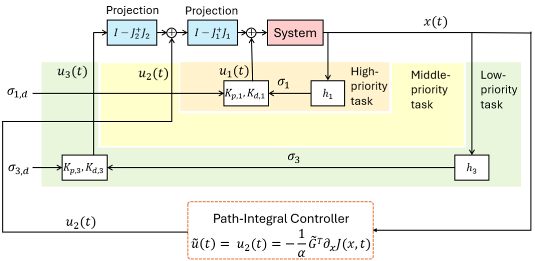

- Integration of null-space projection and path integral control
- In this example, the middle-priority task controller is replaced by a path integral controller

An Example

Suppose a platoon of robots is tasked with three goals:

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- Low-level controllers (such as PID) can't perform optimally
- What if we design an optimal controller using only the path integral method for each robot in the platoon? $\Rightarrow$ scalability challenges
- We propose using low-level controllers for tasks 1 and 3, and apply the path integral controller to the more complex task 2
- The path integral controller optimizes the centroid's movement, while simpler tasks are handled by low-level controllers

Advantages of the Proposed Architecture

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- It allows the hierarchical control architecture to seek the optimal control input
- It achieves a better sample efficiency than naïve applications of path integral controllers to high-DOF systems

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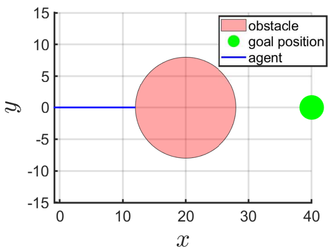
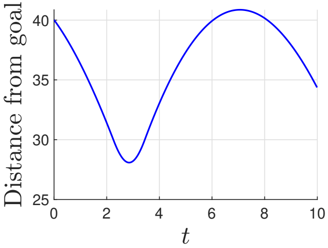
Simulation Results

- A unicycle system
- Two tasks to accomplish in the descending order of importance:
  1. obstacle avoidance
  2. move-to-goal



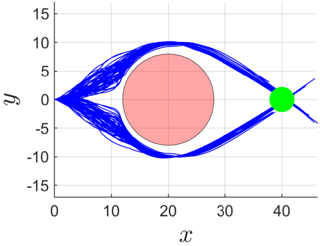
(a) Path followed without path integral controller



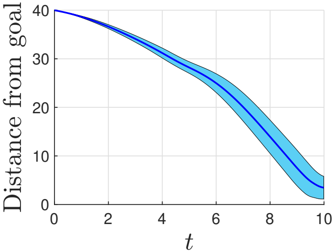
(b) Distance from the goal over time

Figure: Results of single-agent example without the path integral controller

Simulation Results



(a) Paths followed with path integral controller

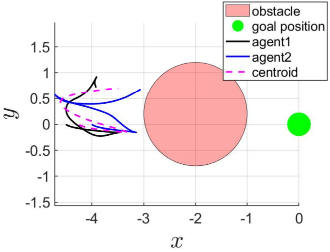


(b) Mean distance from goal $\pm$ standard deviation

Figure: Results of single-agent example using the path integral controller

Simulation Results

- A system of two unicycles
- Three tasks to accomplish in the descending order of importance:
  1. obstacle avoidance
  2. steering the centroid of the robots toward a goal position
  3. maintaining a specific distance between the two unicycles



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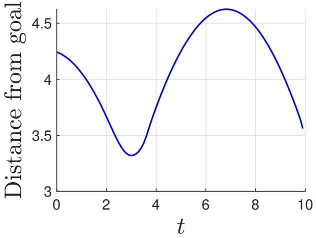
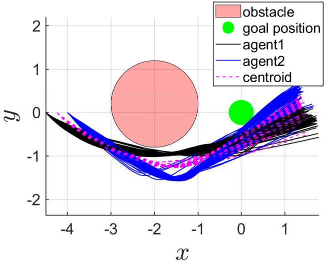


(b) Distance from the goal over time

Figure: Results of two-agents example without the path integral controller

Simulation Results



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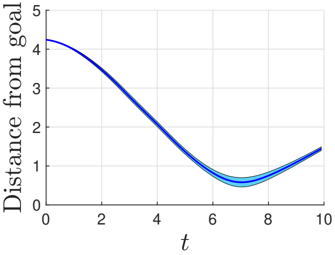

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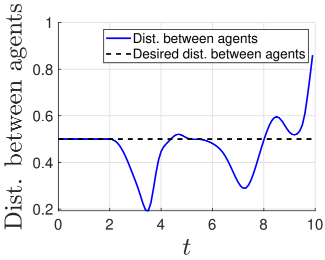
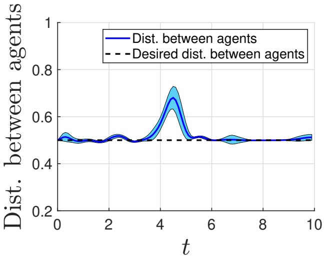


(a) Without path integral controller: Distance between agents



(b) With path integral controller: Mean distance between agents $\pm$ standard deviation

Figure: Distance between agents over time

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Multi-agent red-team blue-team competition

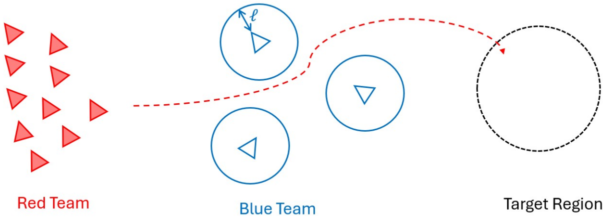

Figure: Differential game extension. The red team attempts to maneuver past the blue team in order to occupy the target region. Meanwhile, the blue team maneuvers in order to block the red team.

Questions?

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